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Optimization of Water-to-Fuel (W/F) Ratios in Cladded Cylinder Arrays

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INTRODUCTION

Often in criticality safety problems, the analyst is concerned about two conditions: Loss of Mass Control and Loss of Moderation Control. Determining and modeling the maximum amount of fuel that can fit in a given container is usually trivial. Determining and modeling the maximum amount of water (or other potential moderator) is usually more difficult.

CLADDED CYLINDER ARRAYS

For arrays of cylinders (fuel pellets, rods, UF_6 gas cylinders, etc.) the maximum amount of fuel that can be stored is found with a tight packed array. However, finding the maximum amount of water that could accumulate between the cylinders is more complicated because the pitch (center-to-center distance) between the cylinders is usually not set (or could be upset) and must be assigned to give the maximum Water-to-Fuel (W/F) volumetric ratio. Figure 1 illustrates a pitched array of cladded cylinders.

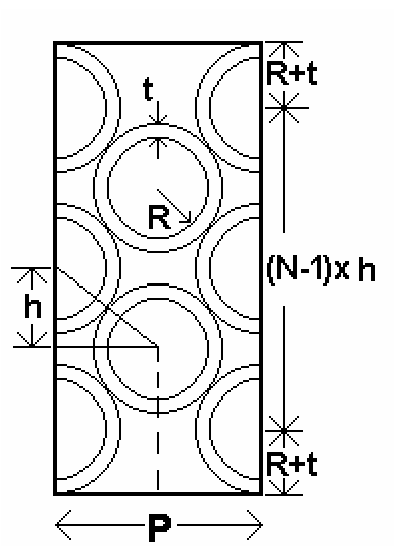

Figure 1. Pitched Array of Fuel Pins Stacked "N" High

This paper derives the maximum amount of interstitial space possible in all orderly stacked cladded cylinder arrays. With given Radius (R) and cladding or cylinder wall thickness (t) the Pitch-to-Radius {P/(R+t)} ratio that will result in the maximum interstitial space is provided.

For example, given that the analysts knows the radius and wall thickness of a given cylinder, he would model a tight packed array with P = 2(R+t) or P/(R+t) = 2. The set pitch between cylinders is simply set so they are touching. To model a loss of moderation control case the pitch must be selected to give the maximum interstitial space (assuming an under-moderated initial condition) that could be filled with the moderator of concern, usually water.

Previously[1], tables have been published of these optimized P/R ratios. This paper expands the previous work by adding the cladding thickness and derives the generalized equation:

Handley/Huffer Pitch Optimization Equation:

$$P^2/(R+t)^2 = 8 - 2/(N-1)^2 + [2/(N-1)^2]\ \mathrm{sqrt}(8N^2 - 16N + 9)$$

with: N = Number of cylinder rows deep
P = Pitch, center-to-center spacing of the cylinders
R = Radius of inner fuel
t = thickness of fuel pellet cladding (plus gap if applicable)

Authors Note: Long before I had actually derived the above general equation, I walked a senior Oak Ridge NCS Engineer, Dick Handley, through the equations and he exclaimed: "We knew it could all be boiled down to one equation!" His comment inspired me to push through to the above expression. After hearing he had recently passed away, I thought the above name was appropriate.

Example

For example; given a three high stack the equation quickly becomes P/(R+t) =sqrt(8 – ½ + ½ * sqrt(33)) = 3.221. Therefore, the model must place cylinders 3.221* (R+t) apart to achieve the maximum amount of moderator between the cylinders.

The required Water-to-Fuel (W/F) volumetric and Hydrogen-to-Fissile isotope (H/X) atomic ratios can then be calculated from the specific problem data. Table 1 provides solved equations for stacks up to 10 high and provides general and infinite cases. Ratios for the tight packed case are also provided for comparison.

Table 1. Maximized W/F Ratios in Stacked Cylinders With Cladding of Thickness (t)

Stack N High	**W/F Relationship** From Geometry	**Math Solution** $\mathbf{P^2/(R+t)^2}$	**Optimum P/(R+t)**	**Maximum W/F***	**Example H/X**	**Tight Pack P=2(R+t) W/F***
2	$\{(2(R+t)+\mathbf{h})P-2\pi(R+t)^2\}/2\pi R^2$	12	3.464	0.654	35.8	0.188
3	$\{(2(R+t)+2\mathbf{h})P-3\pi(R+t)^2\}/3\pi R^2$	(15+sqrt(33))/2	3.221	0.494	27.0	0.160
4	$\{(2(R+t)+3\mathbf{h})P-4\pi(R+t)^2\}/4\pi R^2$	2*(35+sqrt(73))/9	3.111	0.429	23.5	0.145
5	$\{(2(R+t)+4\mathbf{h})P-5\pi(R+t)^2\}/5\pi R^2$	(63+sqrt(129))/8	3.049	0.393	21.5	0.137
6	$\{(2(R+t)+5\mathbf{h})P-6\pi(R+t)^2\}/6\pi R^2$	2*(99+sqrt(201))/25	3.009	0.371	20.3	0.131
7	$\{(2(R+t)+6\mathbf{h})P-7\pi(R+t)^2\}/7\pi R^2$	(143+(17))/18	2.981	0.356	19.5	0.127
8	$\{(2(R+t)+7\mathbf{h})P-8\pi(R+t)^2\}/8\pi R^2$	2*(195+sqrt(393))/49	2.961	0.345	18.8	0.124
9	$\{(2(R+t)+8\mathbf{h})P-9\pi(R+t)^2\}/9\pi R^2$	(255+sqrt(513))/32	2.946	0.336	18.4	0.122
10	$\{(2(R+t)+9\mathbf{h})P-10\pi(R+t)^2\}/10\pi R^2$	2(323+sqrt(649))/81	2.933	0.329	18.0	0.120
Gen	$\{(2(R+t)+(N-1)\mathbf{h})P-N\pi(R+t)^2\}/N\pi R^2$	$8-2/(N-1)^2+[2/(N-1)^2]$*sqrt$(8N^2-16N+9)$		Below		
∞	$(\mathbf{h}P-\pi(R+t)^2)/\pi R^2$	8	2.828	0.273	14.9	0.103

* Each result must be multiplied by $(R+t)^2/R^2$

h = sqrt$(4(R+t)^2-P^2/4)$

General W/F = $\{(2/N\pi)*(P/(R+t))+((N-1)/N\pi)*(P/(R+t))h-1\}*(R+t)^2/R^2$

General Case for tight packed cylinders P = 2(R+t):

W/F = $\{2*\text{sqrt}(3)/\pi + (2/N\pi)*(2-\text{sqrt}(3)) -1\} * (R+t)^2/R^2$

H/X is dependent on Fuel Type, an example equation assuming an oxide fuel submerged in water is:

H/X = (2 * Den (H_2O)/Den(XOn) * At. Wt. (XOn)/ At. Wt. (H_2O)) *** W/F;** w/ X =Fissile Isotope, n=# of oxygen atoms

Table example is 5 wt.% U-235 in UO_2 at 10.96 g/cc in full density Water (with no residual moderation from binders).

COMMERCIAL APPLICATION

To illustrate the effect of selected pitch and resulting Water-to-Fuel Ratio, a case of LEU (5 wt. % U-235 in UO_2) fuel pins was selected with 0.25 wt% residual binder. A 1.0 cm diameter fuel pin with a fuel radius of 0.41 cm and a cladding thickness of 0.09 cm was selected to represent PWR fuel. Two cases are examined in detail.

The first case is an accident condition where portable molybdenum boats (for sintering) were mishandled and stacked five high in a corner. Sprinkler activation and/or pipe break is assumed to have filled and reflected the boats. The boats are assumed to be 25 pellet rows (pitches) wide by 18-inches long. Additionally, the boats are assumed to be located in a corner of Oak Ridge Concrete, 2 foot thick on two sides and the floor.

Per Table 1 the optimum P_{opt}/R=3.049, P_{opt}=1.25 cm, and the maximum W/F Ratio is 0.393. Figure 2 illustrates the results of the stacked molybdenum boat case.

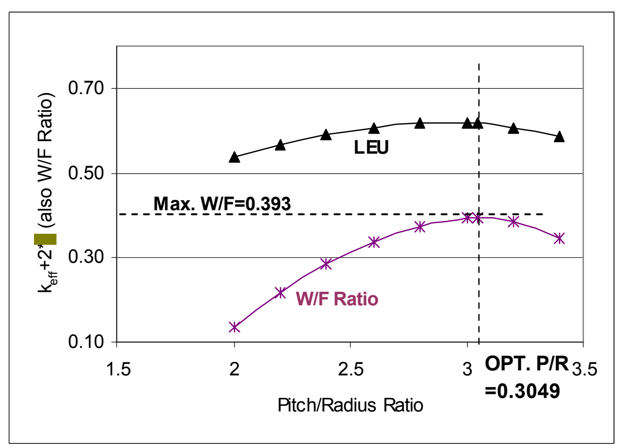

Figure 2. Five Stacked Moly Boats
25 pellets across, 5 pellets deep, with varying Pitch

Per Figure 2 it can be seen that the system reactivity tracks closely to the Water-to-Fuel Ratio. However, the peak reactivity is not exactly the peak system reactivity.

For the second application a bin filled with cladded fuel rods was chosen. The bin was selected to be 25 rows (pitches) wide and 23 rods high, stacked in a single 16 gauge stainless steel bin. The above LEU fuel was used with Zircaloy 2 as the cladding material. Sprinkler activation and/or pipe break is assumed to have filled and reflected the bin. From the presented Pitch Optimization Equation: $P_{opt}/(R+t) = 2.873$, $P_{opt}=1.4364$, and Table 1 provides the Maximum W/F ratio as 0.1231. Figure 3 illustrates the results of the flooded bin case.

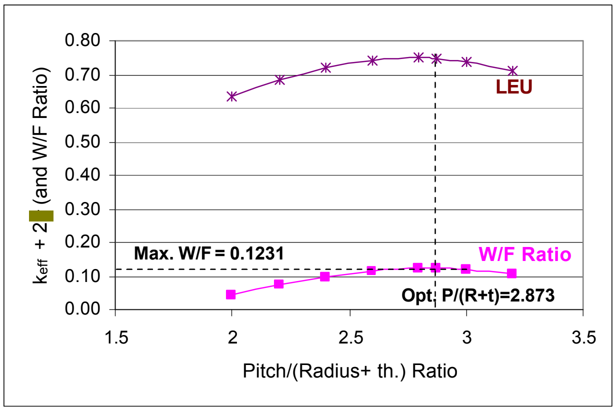

Figure 3. Flooded Fuel Bin with 25 pins wide and 23 pins high, with varying Pitch.

Comparing Figure 3 to Figure 2 shows that the cladding significantly reduces the Water-to-Fuel ratio in these tight packed scenarios.

Figure 3 also demonstrates close tracking between the W/F ratio and the system reactivity. The system reactivity peaks just before the peak Water-to-Fuel ratio in both cases. Notice that as the fuel is spreading out the aerial density is lowering, making for a competing effect.

With the above two cases developed, it was of interest to examine optional fuel types. Table 1 of Reference 2 provides isotopic information for Weapons Grade (WG) and Reactor Grade (RG) Plutonium. Assuming that the WG PuO_2 would be mixed with Natural Uranium and the RG PuO_2 would be mixed with 3 wt.% U-235 in UO_2 at a ratio of 35 wt.% PuO_2 to 65 wt.% UO_2 and 0.25 wt.% binder provides an estimate of potential alternate commercial fuels. Figure 4 illustrates results of executing the five stacked molybdenum boat accident case with all three fuel types.

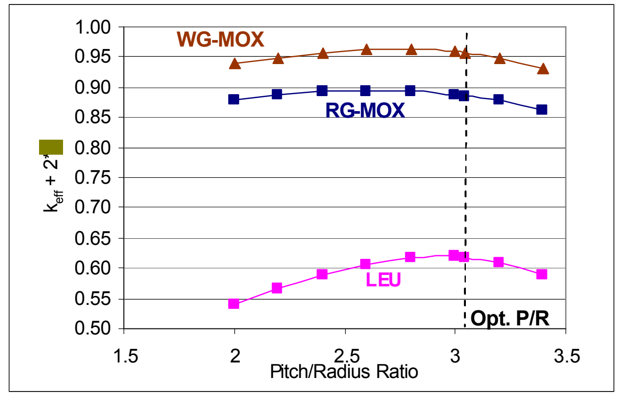

Figure 4. Five Stacked Moly Boats (same model as used in Figure 2) with MOX Fuels.

Figure 4 illustrates that the plutonium containing fuels are also sensitive to the Water-to-Fuel Ratio, although with a different response than LEU fuel.

The bin case presented in Figure 3 was also executed with WG-MOX and RG-MOX, resulting in the same shaped curves as presented in Figure 4.

METHOD

The neutron multiplication calculations presented in Figures 2-4 were performed with KENO VI using CENTRM and the 238-group (ENDF/B-V) cross sections. The input development was automated with PRISM. The calculations were performed on the PC-SCALE5 platform.

RESULTS

Optimization of the pitch has been shown to provide an increase in system reactivity. Both MOX and LEU systems have been shown to be sensitive to moderator intrusion in varying pitched configurations. The analysis will have to determine the effect of optimizing the pitch for each array.

REFERENCES

1. J.E. HUFFER, "Optimization of Water-to-Fuel Ratio in Pellet Arrays"; ANS Transactions, **82**, 170 (2000).
2. Biswas, D. et. al., "Weapons Grade Plutonium Disposition in Pressurized Water Reactors" Nuclear Science and Engineering, Volume 12, Number 1, American Nuclear Society, La Grange Park, Il 60526, September 1995.